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PHYSICAL MECHANISM OF WAVE-PARTICLE RESONANCES IN A CURVED MAGN--ETC(U)
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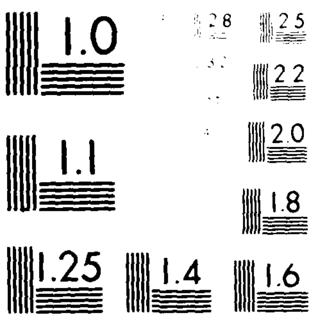


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The physical mechanism of wave-particle resonances in a curved magnetic field is investigated. Specifically, the energy exchange process between a wave and resonant curvature drifting particles is discussed (i.e., $\omega \sim \underset{\sim}{k} \cdot \underset{\sim}{V}_c$ where $V_c = v_\parallel^2/R_c\Omega$ is the curvature drift, $\underset{\sim}{R}_c$ is the radius of curvature of the magnetic field and $\Omega$ is the cyclotron frequency). A general expression for the wave damping/growth rate is derived based upon physical arguments. The theory is applied to the lower-hybrid-drift instability and non-linear consequences are discussed.

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# PHYSICAL MECHANISM OF WAVE-PARTICLE RESONANCES IN A CURVED MAGNETIC FIELD

## I. INTRODUCTION

One of the most important phenomena in plasma physics is the wave-particle resonance.[1] These resonances allow collisionless plasmas to undergo irreversible processes such as energy and momentum exchange, and are crucial to such diverse phenomena as RF heating of laboratory plasmas,[2] isotope separation[3] and anomalous transport of particles, momenta and energies across boundary layers.[4] The classic work of Landau demonstrated that in an unmagnetized collisionless plasma, an electron plasma wave is damped by electrons that are in resonance with the wave; those particles which have a velocity such that $\omega \sim kv$ .[1] In magnetized plasmas, Landau resonances can also occur when $k_\parallel = \underset{\sim}{k}\cdot\underset{\sim}{B}/B$ , the component of the wave vector parallel to the magnetic field B, is non-zero (for $\omega \sim n\Omega - k_\parallel v_\parallel$ with $\Omega$ the cyclotron frequency and n an integer). On the other hand, flute modes with $k_\parallel = 0$ are important in a number of physical phenomena, including plasma instabilities and radio frequency heating. If the confining magnetic field is inhomogeneous, a cross-field, wave-particle resonance can occur for particles undergoing a magnetic drift, i.e., $\omega \sim \underset{\sim}{k}\cdot\underset{\sim}{V}_B$ where $\underset{\sim}{V}_B$ may be due to the spatial inhomogeneity of the magnetic field ($\nabla B$ drift) and/or to the curvature of the magnetic field (curvature drift). These drifts are described as follows.

In an inhomogeneous magnetic field, the two important magnetic drifts that can exist[5] are the $\nabla B$ drift

$$\underset{\sim}{V}_{\nabla B} = \frac{v_\perp^2}{2\Omega}\,\frac{(\underset{\sim}{B}\times\nabla)B}{B^2} \tag{1}$$

and the curvature drift

$$\underset{\sim}{V}_c = \frac{v_\parallel^2}{\Omega}\,\frac{\underset{\sim}{B}\times(\underset{\sim}{B}\cdot\nabla)\underset{\sim}{B}}{B^3} \tag{2}$$

where $\Omega = eB/mc$ is the cyclotron frequency. If both types of

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drifts exist and the magnetic field is produced by an external source (so that there are no plasma currents) then the total drift is given by

$$\underset{\sim}{V}_B = (v_\perp^2/2 + v_\parallel^2)\,\frac{(\underset{\sim}{B} \times \nabla)B}{\Omega B^2} \tag{3}$$

Two salient features of $\underset{\sim}{V}_B$ are the following. First, $\underset{\sim}{V}_B$ is a microscopic drift so that individual particles acutally undergo a drift proportional to $v_\perp^2$ and/or $v_\parallel^2$ . It is this property that allows a wave-particle resonance to occur; unlike diamagnetic drifts (due to $\nabla n$ or $\nabla T$) which are macroscopic drifts. Second, $\underset{\sim}{V}_B$ is charge dependent so that electrons and ions drift in opposite directions. This implies that only one species of particles can be resonant with a given wave. Recently, we have investigated the physical processes underlying the wave-particle resonance due to a $\nabla B$ drift ($\omega \sim \underset{\sim}{k}\cdot\underset{\sim}{V}_{\nabla B}$ ).[6,7] In this paper we complete our study of magnetic drift resonances by focussing on curvature drift resonances ($\omega \sim \underset{\sim}{k}\cdot\underset{\sim}{V}_c$). Of course, both drifts exist in many plasmas of interest but we will ignore the $\nabla B$ drift for pedagogical purposes.

The curvature drift can be viewed as arising from the centrifugal force acting on the particle in its rest frame.[8] This is shown in Fig. 1. Here, $\underset{\sim}{B} = B_x(z)\,\hat{e}_x + B_z\,\hat{e}_z$, $F_{cp}$ is the centripetal force acting on the particle as it follows the curved field line and $F_{cf} = -F_{cp}$ is the centrifugal force felt by the particle in its rest frame. We restrict our attention to the xy plane and take

$$\underset{\sim}{F}_{cp} = \frac{m v_\parallel^2}{R_c}\,\hat{e}_x = -\underset{\sim}{F}_{cf} \tag{4}$$

where $R_c$ is the radius of curvature of the field at the origin. The curvature drift can then be written as

$$\underset{\sim}{V}_c = \frac{1}{m\Omega}\,\frac{\underset{\sim}{F}_{cf} \times \underset{\sim}{B}}{B} = \frac{v_\parallel^2}{R_c\Omega}\,\hat{e}_y \tag{5}$$

so that ions (+e) drift in the +y-direction and electrons (-e) drift in the -y-direction as shown in Fig. 1. From Eq. (5) it is clear that a wave travelling in the +y-direction can be in phase with ions if $\omega \sim k_y v_{\parallel r}^2 / R_c \Omega$ where $v_{\parallel r}$ is the parallel velocity necessary for resonance. Also, since $V_c$ depends upon $v_{\parallel}^2$, both $+v_{\parallel r}$ and $-v_{\parallel r}$ particles can be in resonance.

We point out that in some previous treatments of the curvature drift, the centrifugal force $F_{cf}$ is replaced by a gravitational force such that $\underset{\sim}{g} = -2T/mR_c \, \hat{e}_x$ (again, we restrict our attention to the xy plane).[9,10] The curvature drift then becomes $\underset{\sim}{V}_c = g/\Omega \, \hat{e}_y$. For this representation of $V_c$, wave-particle resonances cannot occur and only the non-resonant behavior of the particles due to $V_c$ is considered. That is, all of the particles are drifting at $V_c$. Thus, by simply considering $\underset{\sim}{V}_c = g/\Omega \, \hat{e}_y$, potentially important wave-particle resonances are neglected.

The purpose of this paper is to discuss the physical mechanism of wave-particle resonances in a curved magnetic field. The scheme of the paper is as follows. In the next section we present a discussion of the energy exchange process that occurs in this type of resonance. In Section III we present a derivation of the damping/growth rate due to this resonance based upon the physical arguments set forth in Section II. In the final section we discuss an application of this work to the lower-hybrid-drift instability and nonlinear effects.

## II. PHYSICAL DESCRIPTION OF THE WAVE-PARTICLE RESONANCE

To elucidate the energy exchange mechanism of wave-particle resonances in a curved magnetic field we consider the following simplified model. A slab geometry is used with $\underset{\sim}{B} = B_x(z)\,\hat{e}_x + B_z\,\hat{e}_z$ as shown in Fig. 1. The plasma is assumed to be homogeneous. An electrostatic wave is imposed on the system such that $\underset{\sim}{E} = \delta E \sin(ky - \omega t)\,\hat{e}_y$ with $\omega \ll \Omega$ and $kr_L \ll 1$ where $\Omega$ is the cyclotron frequency and $r_L$ is the mean particle Larmor radius. For this wave, which is propagating in the +y-direction (see Fig. 2), only the positively charged species can resonate with it so we limit our discussion to the ions. Resonant ions satisfy the phase matching condition

$$V_{ph} = \frac{\omega}{k} \simeq V_c^r = \frac{v_{\parallel}^2}{R_c \Omega} \tag{6}$$

where $V_c^r$ is the curvature drift, $v_{\parallel r}$ is the parallel velocity of the particle in resonance and $R_c$ is the local radius of curvature of the field. Note that in Fig. 2 we have translated our coordinate system to the wave frame.

Resonant particles at $y = y_1$ see a constant electric field $\underset{\sim}{E} = \delta E\,\hat{e}_y$ and $E \times B$ drift in the +x-direction with a velocity $\delta V_E = c\delta E/B$. Since these particles are moving in the same direction as the centripetal force, the centripetal force is doing work on the particles and therefore is increasing their parallel energy. Thus, these particles absorb energy from the wave and cause damping. Alternatively, the increase in the particle's parallel energy can be obtained from conservation of angular momentum, $L = v_{\parallel} R = \text{const}$. As the particles move in the +x-direction, the radius of rotation decreases so that $v_{\parallel}$ increases to conserve L.

On the other hand, resonant particles at $y = y_2$ see a constant electric field $\underset{\sim}{E} = -\delta E\,\hat{e}_y$ and $E \times B$ drift in the -x-direction with a velocity $\delta V_E = c\delta E/B$. Since these particles are moving opposite to the centripetal force, they must expend energy to overcome this force and hence, decrease their parallel

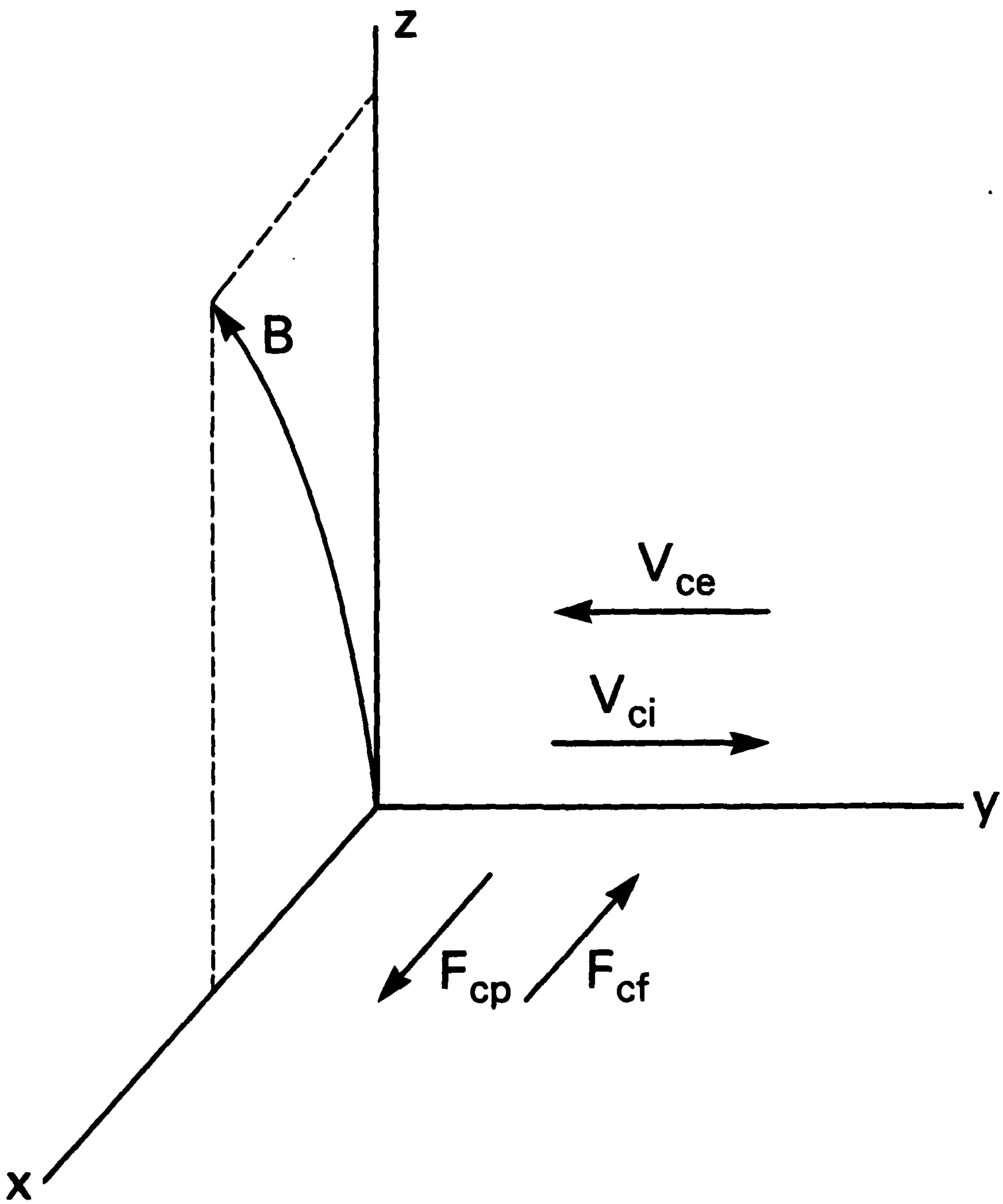


Fig. 1 — Slab geometry used in the analysis with $\underline{B} = B_x \hat{e}_x + B_z \hat{e}_z$. Here, $F_{cp}$ is the centripetal force, $F_{cf}$ is the centrifugal force and $\tilde{V}_{c\alpha}$ is the curvature drift of species $\alpha$.

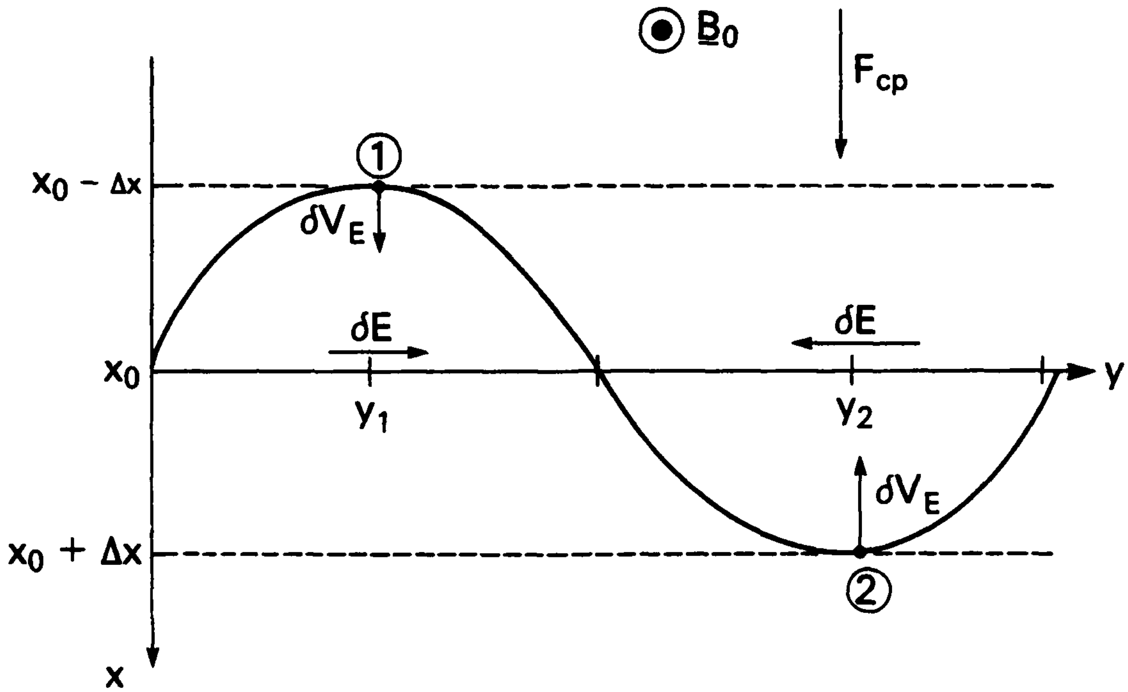


Fig. 2 — Electrostatic wave in a plasma containing a curved magnetic field (wave frame)

energy. Thus, these particles lose energy to the wave and cause wave growth. Again, this can be seen from conservation of L. As particles move in the -x-direction, the radius of rotation increases so that $v_\parallel$ decreases to conserve L.

Clearly, if there are an equal number of particles at position (1) and (2), then there is no net energy exchange between the wave and the resonant particles. However, this is not the situation in general as we now show. The key point is that the number of resonant particles is locally proportional to $f(v_\parallel,x)$, the particle distribution function. As in the case of Landau damping, unequal numbers of particles participate in energy gain/loss transfer to the wave if $\partial f/\partial v_\parallel \neq 0$ (or, in this case also $\partial f/\partial x \neq 0$). We now expand upon this point.

We consider resonant particles ($v_\parallel = v_{\parallel r}$) at $x = x_o$ and at some time $t_o + \Delta t$ , i.e., $f(x_o,v_{\parallel r},t_o+\Delta t)$. We ask the question, where were these particles at an earlier time $t = t_o$? A portion of these particles were at position (1) ($x = x_o - \Delta x$, $y = y_1$) with a velocity $v_\parallel = v_{\parallel r} - \Delta v_\parallel$. In a time $\Delta t$, these particles move a distance $\Delta x$ and increase their velocity by $\Delta v_\parallel$ to arrive at $x = x_o$ and $v_\parallel = v_{\parallel r}$ . Thus, these particles are described by $f(x_o-\Delta x,v_{\parallel r}-\Delta v_\parallel,t_o)$ and abosrb energy from the wave, i.e., lead to damping. The rest of the particles were at position (2) ($x = x_o + \Delta x$, $y = y_2$) with a velocity $v_\parallel = v_{\parallel r} + \Delta v_\parallel$ . In a time $\Delta t$ these particles also move a distance $\Delta x$ but decrease their velocity by $\Delta v_\parallel$ to arrive at $x = x_o$ and $v_\parallel = v_{\parallel r}$. Thus, these particles are described by $f(x_o+\Delta x,v_{\parallel r}+\Delta v_\parallel,t_o)$ and give energy to the wave, i.e., lead to growth. Thus, if $f(x_o-\Delta x,v_{\parallel r}-\Delta v_\parallel,t_o) > f(x_o+\Delta x,v_{\parallel r}+\Delta v_\parallel,t_o)$ then more particles absorb energy from the wave than give energy to the wave and wave damping results. Neglecting any spatial dependence on f, i.e., no density or temperature gradients, this means that $\partial f/\partial v_\parallel < 0$ for wave damping, as in the Landau resonance. Alternatively, if $\partial f/\partial v_\parallel > 0$ then wave growth can result. Finally, we note that resonance particles exist at both $v_\parallel = v_{\parallel r}$ and $v_\parallel = - v_{\parallel r}$ since $V_c^r \propto v_\parallel^2$ . For symmetric particle distribution functions in $v_\parallel$ this introduces a factor of 2 in the damping/growth rate (see Fig. 3).

Thus, energy transfer can occur in wave-particle resonances due to magnetic field curvature drifts because the wave electric field cause particles to E × B drift in the same direction or opposite to the centripetal force acting on the particle as it moves along the curved field line. Alternatively, the energy transfer mechanism can be viewed as conservation of angular momentum, $L = v_\parallel R$. We now derive a specific expression for the damping/growth rate due to a magnetic curvature drift-wave resonance based on this physical picture.

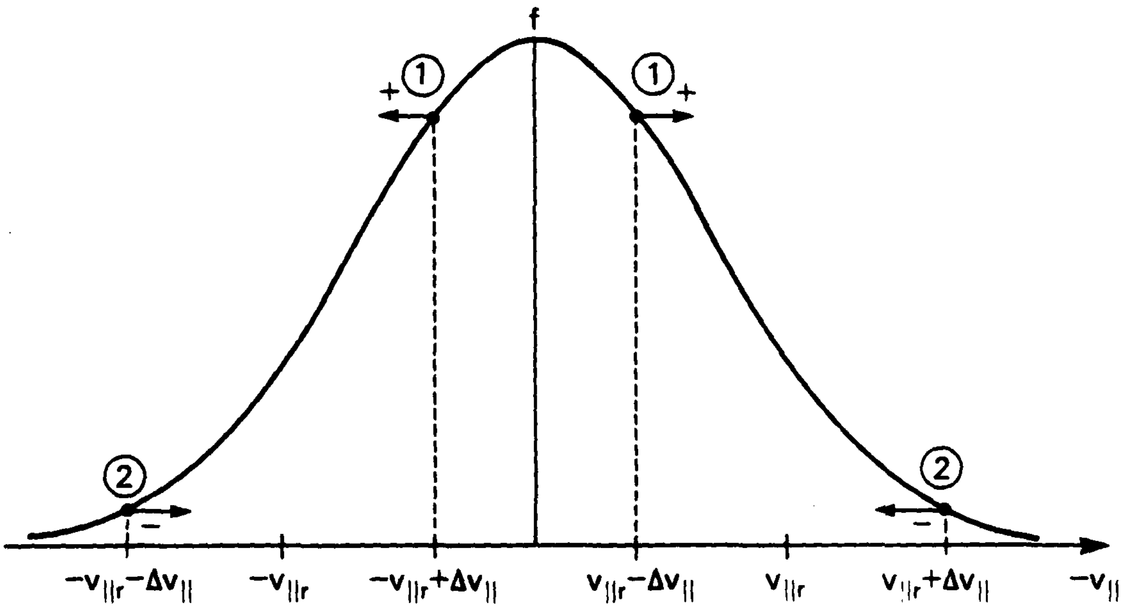


**Fig. 3 — Particle distribution function indicating resonant parallel velocities. Here, $v_\parallel = v_{\parallel r} - \Delta v_\parallel$ and $v_\parallel = -v_{\parallel r} + \Delta v_\parallel$ are the resonant velocities at position (1), and $v_\parallel = v_{\parallel r} + \Delta v_\parallel$ and $v_\parallel = -v_{\parallel r} - \Delta v_\parallel$ are the resonant velocities at position (2). The + signs indicate that particles at position (1) gain energy, while the − signs at position (2) indicate that the particles lose energy.**

## III. PHYSICAL DERIVATION OF THE WAVE DAMPING/GROWTH RATE

We calculate the damping/growth of the wave due to the particle curvature drift resonance using energy conservation. Total energy conservaton can be expessed as

$$\dot{\Sigma}_p + \dot{\Sigma}_w = 0 \tag{7}$$

where $\Sigma_p$ and $\Sigma_w$ are the particle and wave energy density, respectively, and the dot indicates a time differentiation. The wave energy density contains both the electrostatic field energy and the sloshing energy of the particles. Our method is similar to that of Meade,[11] who calculated the growth rate of the universal drift instability.

The wave energy density of an electrostatic wave in a lossless medium is given by

$$\Sigma_w = \frac{|\delta E|^2}{8\pi} [\omega \; \partial D/\partial\omega] \tag{8}$$

where D is the dielectric function. Assuming that the perturbed field varies as exp $(i\omega t)$ where $\omega = \omega_r + i\gamma$ we obtain

$$\dot{\Sigma}_w = \gamma \frac{|\delta E|^2}{4\pi} [\omega \; \partial D/\partial\omega] \tag{9}$$

The change in energy of a single particle is given by

$$\Delta W_p = \underset{\sim}{F} \cdot \Delta \underset{\sim}{x} = F_x \; \Delta x \tag{10}$$

where $F_x = mv_{\parallel r}^2/R_c$ is the centripetal force acting on the particle locally at $(x,z) = (0,0)$. We then obtain

$$\Delta W_p = \frac{mv_{\parallel r}^2}{R_c} \Delta x \tag{11}$$

Since the particle velocity is $\delta \underset{\sim}{V}_E = c\delta E/B \; \hat{e}_x$ we find that

$$\Delta W_p = \frac{mv_{\parallel}^2}{R_c} \frac{c\delta E}{B} \Delta t \tag{12}$$

or

$$\dot{W}_p = \frac{mv_{\parallel r}^2}{R_c} \frac{c\delta E}{B} \tag{13}$$

Noting that $V_c^r = v_{\parallel r}^2/R_c\Omega$, we rewrite Eq. (13) as

$$\dot{W}_p = e\ \delta E\ V_c^r \qquad (= \underset{\sim}{j}\cdot\underset{\sim}{E}/n) \tag{14}$$

so that the time rate of change of a single particle's energy is related to its Joule heating since $\underset{\sim}{j} = en\underset{\sim}{V}_c$ for the curvature drift particles. From Eq. (14) we point out that for $\delta E > 0$ the particle gains energy while for $\delta E < 0$ it loses energy.

The time rate of change of the total particle energy density $\dot{\Sigma}_p$ is found by calculating the change of energy of all the resonant particles in a time $\Delta t$. We integrate the equilibrium distribution function over the range $v_{\parallel r} - \Delta v_{\parallel r}$ to $v_{\parallel r} + \Delta v_{\parallel r}$ where $\Delta v_{\parallel r}$ is the spread in $v_\parallel$ for a particle to remain in resonance with the wave. Making use of Eq. (14) we find that

$$\dot{\Sigma}_p = \frac{1}{2}\, e\delta E V_c^r \Big( [f(v_{\parallel r}-\Delta v_\parallel, x_o-\Delta x) - f(v_{\parallel r}+\Delta v_\parallel, x_o+\Delta x)]$$
$$+ [f(-v_{\parallel r}+\Delta v_\parallel, x_o-\Delta x) - f(-v_{\parallel r}-\Delta v_\parallel, x_o+\Delta x)] \Big)\ \Delta v_{\parallel r} \tag{15}$$

where $\Delta v_\parallel$ and $\Delta x$ represent the change in parallel velocity and position in a time $\Delta t$, respectively, and the $\phi$ and $v_\perp$ integrations have been performed. In Eq. (15) we only consider a density inhomogeneity since a temperature inhomogeneity is usually only important for flute modes when $kr_L \approx 1$ and we are assuming $k^2 r_L^2 << 1$. [9]

From Eq. (15) we note that the resonant particles in the first bracket occur at $v_\parallel = +v_{\parallel r}$ while those in the second bracket occur at $v_\parallel = -v_{\parallel r}$ (see Fig. 3). Moreover, the particles at $(x_o - \Delta x,\ v_{\parallel r} - \Delta v_\parallel)$ and $(x_o - \Delta x,\ -v_{\parallel r} + \Delta v_\parallel)$ increase their energy (+ sign in Eq. (15)) and particles at $(x_o + \Delta x,\ v_{\parallel r} + \Delta v_\parallel)$ and $(x_o + \Delta x,\ -v_{\parallel r} - \Delta v_\parallel)$ decrease their energy (- sign in Eq. (15)). We can rewrite Eq. (15) as

$$\dot{\Sigma}_p = e\delta E V_c^r \left( \left[ -\left(\Delta v_\parallel \frac{\partial f}{\partial v_\parallel} + \Delta x \frac{\partial f}{\partial x}\right)_{v_{\parallel r}, x_o} \right.\right.$$

$$\left.\left. + \left(\Delta v_\parallel \frac{\partial f}{\partial v_\parallel} - \Delta x \frac{\partial f}{\partial x}\right)_{-v_{\parallel r}, x_o} \right] \right) \Delta v_{\parallel r} \qquad (16)$$

Assuming $f$ is symmetric in $v_\parallel$, i.e.,

$$(\partial f/\partial v_\parallel)_{v_{\parallel r}} = -(\partial f/\partial v_\parallel)_{v_{\parallel r}} \qquad (17)$$

we finally obtain

$$\dot{\Sigma}_p = -2e\delta E V_c^r \quad \left[\Delta v_\parallel \frac{\partial f}{\partial v_\parallel} + \Delta x \frac{\partial f}{\partial x}\right]_{v_{\parallel r}, x_o} \quad \Delta v_{\parallel r} \qquad (18)$$

so that the dependence of $\dot{\Sigma}_p$ on the slope of the distribution function is evident.

We now compute $\Delta x$, $\Delta v_\parallel$ and $\Delta v_{\parallel r}$ as follows. The distance a particle moves in a time $\Delta t$ is simply $\Delta x = \delta V_E \Delta t = (c\delta E/B)\Delta t$. The change is a particle's parallel energy is found from Eq. (12) and the definition $\Delta W_p = \Delta(mv_\parallel^2/2)$. We find that

$$\Delta v_\parallel = \frac{v_{\parallel r}}{R_c} \frac{c\delta E}{B} \Delta t \ . \qquad (19)$$

This can also be obtained from the conservation of angular momentum, i.e., $L = v_\parallel R$ = constant and using $\partial R/\partial t = -\delta V_E$. Finally, $\Delta v_{\parallel r}$ is determined by noting that for a particle to remain in resonance with a wave, we require

$$\Delta V_c^r < \frac{\pi}{k\Delta t} \qquad (20)$$

That is, particles will only be in resonance $(\omega \sim kV_c^r)$ as long as they do not move more than a half-wavelength in a time $\Delta t$. Making use of the definition of $V_c^r$ (Eq. (6)) we find that

$$\Delta v_{\parallel r} = \frac{\pi}{2} \frac{\Omega R_c}{v_{\parallel r} k \Delta t} . \qquad (21)$$

Substituting $\Delta x$, $\Delta v_\parallel$ and $\Delta v_{\parallel r}$ into Eq. (18), we arrive at

$$\dot{\Sigma}_p = - \frac{\pi e^2 \omega}{m k^2} \left[ \frac{\partial f}{\partial v_\parallel} + \frac{R_c}{v_\parallel} \frac{\partial f}{\partial x} \right]_{v_{\parallel r}, x_o} |\delta E|^2 . \qquad (22)$$

The damping/growth rate due to the curvature drift resonance is now found from Eqs. (7), (9) and (22). We find that

$$\frac{\gamma_c}{\omega} = \pi \frac{\omega_p^2}{k^2} \left[ \frac{\partial f}{\partial v_\parallel} + \frac{R_c}{v_\parallel} \frac{\partial f}{\partial x} \right]_{v_{\parallel r}, x_o} \left[ \omega \frac{\partial D}{\partial \omega} \right]^{-1} \qquad (23)$$

where $\omega_p^2 = 4\pi n e^2/m$, $\Omega = eB/mc$ and we have normalized f to $n_o = n(x=x_o)$. Thus, Eq. (23) is a general expression for the damping/growth rate, $\gamma_c$, of an electrostatic wave propagating across a magnetic field due to a wave-particle resonance arising from magnetic field curvature drifting particles. Interestingly, for $\partial f/\partial x = 0$, Eq. (23) is the same as for the Landau resonance except for an additional factor of 2 due to the two resonances at $\pm v_{\parallel r}$.[13,14]

## IV. DISCUSSION

A substantial amount of literature exists on the physics of Landau wave-particle resonances (i.e., $\omega \sim k_\parallel v_\parallel$),[12,13] yet there is little discussion of cross-field, wave-particle resonances due to magnetic drifts. The two important cross-field magnetic drifts are the $\nabla B$ drift and the magnetic curvature drift. Recently, we have investigated the physics of the wave-particle resonance for $\nabla B$ drifting particles.[6,7] In this paper we focus our attention on the energy exchange between waves and resonant curvature drifting particles ($\omega \sim \underset{\sim}{k} \cdot \underset{\sim}{V}_c$). The curvature drift can be viewed as arising from the centrifugal force acting on the particle in the particle's rest frame. That is, $\underset{\sim}{V}_c \propto \underset{\sim}{F}_{cf} \times \underset{\sim}{B}$ where $F_{cf} = -mv_\parallel^2/R_c$ and $R_c$ is the radius of curvature of the field line. Curvature drifting particles, resonant with a wave, see a constant electric field which causes them to $E \times B$ drift in the same direction or opposite to the centripetal force acting on them, depending on the phase of the wave. The particle's energy then changes by an amount $\Delta W_p = \Delta(mv_\parallel^2/2) = F_{cp}\Delta x$ where $F_{cp} = mv_\parallel^2/R_c$ is the centripetal force and $\Delta x = (c\delta E/B)\Delta t$. Depending on the relative number of particles absorbing energy from or giving energy to the wave, which is a function of the slope of the particle distribution function in $v_\parallel$ and x space, wave damping or growth can occur. Alternatively, the energy exchange process can be viewed in terms of conservation of angular momentum, $L = v_\parallel R = $ const. The $E \times B$ drift of the resonant particles causes the particles to move to smaller or larger radii of rotation (R) depending on the phase of the wave. Thus, $v_\parallel$ increases or decreases accordingly in order to conserve L. A general equation for the damping/growth rate has been derived based upon these physical arguments, Eq.(23). As an application of this theory we now consider the lower-hybrid-drift wave instability.

The dielectric function for the lower-hybrid-drift mode is given by[4]

$$D = \frac{\omega_{pe}^2}{\Omega_e^2} + \frac{2\omega_{pi}^2}{k^2 v_i^2} (1 - kV_{di}/\omega) \tag{24}$$

where $V_{di} = (v_i^2/2\Omega_i)\, \partial \ln n/\partial x$ and we have assumed $T_e \ll T_i$, $V_{di} \ll v_i$ and $\omega_{pe}^2 \gg \Omega_e^2$ for simplicity. This mode propagates in the direction of the ion diamagnetic drift. In writing Eq. (24) we have ignored the ion Landau resonance to highlight the curvature resonance. Since this instability requires $\gamma > \Omega_i$, the ions behave as unmagnetized particles and cannot participate in the type of resonance discussed in this paper. On the other hand, field line curvature does effect the ion equilibrium drift.[14] We choose the field line curvature to be such that an electron-wave resonance can occur. The electron distribution function is chosen to be

$$f_e = \frac{n(X)}{n_o} (1/\pi v_e^2)^{1/2} \exp(-v_\parallel^2/v_e^2) \tag{25}$$

where $v_e^2 = 2T_e/m_e$ and $X = x - v_y/\Omega_e$ is a constant of the motion. Expanding X about $x_o$ we obtain

$$\frac{\partial f}{\partial v_\parallel} = - \frac{2v_\parallel}{v_e^2} f \quad ; \quad \frac{\partial f}{\partial x} = \varepsilon_n f \tag{26}$$

where $\varepsilon_n = \partial \ln n/\partial x$. Using Eqs. (23) and (26), we find that

$$\frac{\gamma_c}{\omega} = -\sqrt{\pi}\, \frac{T_i}{T_e} (1 - \varepsilon_n R_c/2\zeta^2)\, \zeta \exp(-\zeta^2) \tag{27}$$

where $\zeta = v_{\parallel r}/v_e = (\omega/kV_{ce})$ and $V_{ce} = v_e^2/R_c\Omega_e$. If we define $R_c = 1/\varepsilon_B$ then Eq. (27) is consistent with the results of Rahal and Gary.[15] For the curvature damping to be significant one requires that $\zeta^2 \sim 1$ which places the following condition on $R_c$. We take $\omega/k \sim V_{di}$ so that we require $V_{di} \sim V_{ce}$ which leads to $R_c \sim 2L_n(T_e/T_i)$ where $L_n = (\partial \ln n/\partial x)^{-1} = 1/\varepsilon_n$. Thus, the radius of curvature of the magnetic field lines has to be comparable to the scale length of the density gradient for the electron curvature drift-wave resonance to be a significant damping mechanism for the lower-hybrid-drift wave instability. It should be noted that this electron-wave resonance always leads to damping. For the geometry shown in Fig. 1, we require $\varepsilon_n < 0$ for an electron-wave resonance to occur because $\underset{\sim}{V}_{di} = -V_{di}\, e_y$; from Eq. (27) it is clear that $\varepsilon_n < 0$ leads to damping. On the other hand, if $\varepsilon_n > 0$, (which could conceivably lead to $\gamma_c > 0$), no resonance can occur because then the wave and drifting electrons are not moving in the same direction. In fact, this is the situation considered by Rahal and Gary[15] so that they did not find any resonance damping.

Finally, we note that this resonance may lead to spatial trapping of particles as in the case of the $\nabla B$ drift-wave resonance. That is, as particles drift in the x-direction they eventually lose resonance with the wave; they either drift faster or slower than the wave. This can arise because $v_\parallel$ changes, or because $R_c$ and $\Omega$ are functions of x. The particles then "circle around" and become resonant with the wave in its opposite phase. Thus, they drift in the opposite direction and become trapped. However, it is unclear that such a process can occur since it requires $\tau_{tr} \ll \tau_b$, where $\tau_{tr}$ is the spatial trapping period and $\tau_b$ is the bounce period in a curved field (e.g., mirroring period), which is probably difficult to satisfy in most plasmas of interest.

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